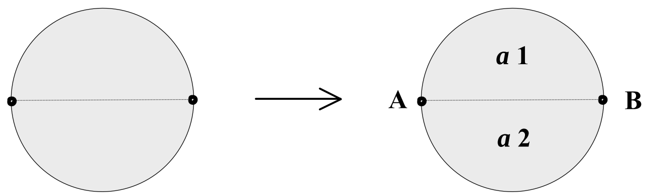
When talking with 5-and-6-year-old students about the circle and what they see when folding it, common first grade words are used. Common usage words are generalizations with different coloration depending on the context when learned. This goes for any age. Giving an accurate word expression to our experiences sometimes requires a lot of words. Language is one of our first tools in abstracting expression from the experiential context. Algebra is a language that is more precise than words. It is a code for words that describe observable relationships. Children learn multiple languages easily when they are young. Algebra is another language. The basics can be learned through folding the circle. It is good fun in abstract thinking for first grade students and they learn a lot about geometry and mathematics. All languages are a tool for giving expression to our experiences and perceptions, and for sharing those with others.

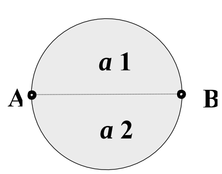


<u>Above</u>. It goes like this; we fold a circle in half (pp.36,71). The diameter is the first line in the circle. It has two end points. We give the individual points names, calling one point Albert, and the other point Bob. To shorten the writing of the names we can just say point A and point B. By naming the points we have named the line. It is between point A and point B, so we call the line AB. We have three names, one for each part from only two letters. We can call the circle O, that's easy. Since line AB divides O in half there are two shapes of areas, they also need names. We decide on area 1 and area 2. To make it short we say A1 and A2. Having already used the big A we use a little a; *a1* and *a2*. (This is about seeing differences.) We talk about this until someone sees we have a line BA. Usually we have talked about movement going in two directions and they see the difference between AB and BA is direction. There is one path, line, going in two directions. So we see AB equals BA, and BA is the same as AB. By using two arrows in opposite directions between AB and BA we can show they are the same but with different directions, **AB** $\rightleftarrows$ **BA.** Or just make two short lines **AB=BA**. We can then talk about the symmetry of the fold in a code language.

Two more paths are observed between point A and point B. Each one of these goes around the circle in two different directions. Now, this presents a problem. We must be able to show the difference between the three different pathways and six different directions. Each half of the circle shows a curved path. So for the top path we can put a curved line over the path AB, and the same for BA. For the bottom path a curved line is put under the AB and BA, just like the circle shows. To make it clear we can put a straight line over AB and BA to show the straight path. We had to all agree on this so there was no confusion about what paths and which directions we mean. We now have $\overline{AB}$, $\overline{BA}$, $\overset{\frown}{AB}$, $\overset{\frown}{BA}$, $\underset{\smile}{AB}$, $\underset{\smile}{BA}$

We have now given names to all three paths (two major arcs and a diameter), and six directions (movement between points), two points (vertices), one line (bisector), two areas (major sectors) and one circle (the context). Compare this information to one path and two directions in the straight-line equation AB=BA that is usually given as a commutative function in Algebra books.

<u>Below</u>. The circle is infinitely symmetrical and by folding it one line of symmetry is formed. One half is equal to the other half. All parts come in pairs except the diameter, which has two different directions. We have set up our symbols in pairs to show they are equal. This reflects the dual nature of folding the circle.



$\overline{AB} = \overline{BA}$	$\overset{\frown}{AB} = \overset{\frown}{BA}$	$\underset{\smile}{AB} = \underset{\smile}{BA}$	$a1 = a2$
$\overline{BA} = \overline{AB}$	$\overset{\frown}{BA} = \overset{\frown}{AB}$	$\underset{\smile}{BA} = \underset{\smile}{AB}$	$a2 = a1$

The first fold generates two points. They are the same except they are in different places, so we call them similar. There are two directions because we can start from either point. The two areas of the circle are the same but in different places, so they are also similar. We can say the same about the curved paths, they are the same in different places. Parts and functions that are the same are similar when in different locations. The abstract symbols look the same. When we show parts are the same we put an equal sign (=) between them. If we want to show they are not the same we can cross out the equal sign by putting a diagonal line through it. $\overline{AB} \neq \overset{\frown}{AB}$ shows they are not the same, they are unequal.

Symbols make it easy to add and subtract parts and to rearrange them, which is what happens between multifunctional parts. With symbols we can observe more information and often more accurately than when using words. At this point in development we have no rules other than those on which the class agrees. We play a lot with seeing how many different ways we can arrange the parts to make the two sides equal, or unequal. The more folds we make the more parts there are. It gets confusing so we have to expand our code language. That is when we add more name symbols to identify different parts and find new relationships. Then we can expand the game by discovering different kinds of equations of similarities and differences. It helps students to observe and think in a different way when we use name symbols instead of common language.

How many different combinations of parts can you describe by using the code we have developed so far? This is good to do with the entire class at the board where each student can contribute what they see. This way the entire class can agree if each equation follows the rules. Remember these rules are what the students have agreed upon. When symbols are used to represent the parts in the circle, we can move the parts around which we cannot do in the circle itself. Representations of parts do not have the same restrictions as actual parts, but then symbols are only names we have given to the parts to talk about them. Symbols always represent something else. That stretches our imagination as we make connections to collections of other objects to see if they work in the same way. The use of symbols allows us to see things in space that we would not see otherwise, to see patterns and arrangements that we would normally miss. Symbols help us to make connections, as we have seen with numbers. The usefulness of any language is in the consensual, contextual meaning we share, and the value we gain in collectively sharing our observations and experiences with others.

Below. Some simple equations to be explored, always in reference to the circle O, they have folded.

AB = BA	AB +BA = $a1$	O − $a1$ = $a2$
2AB = 2BA	AB + BA = $a2$	$a2 - \overset{\frown}{BA} = \overline{AB}$
BA = BA	2AB + 2BA = $a1 + a2$	$\overset{\frown}{AB} + \underset{\smile}{BA} = O$
AB + 2 = BA + 2	$a1 + a2$ = O	$a1$ = O − $a2$
AB + 1 $\neq$ BA + 2	$a1 + a2 = \overset{\frown}{AB} + \underset{\smile}{BA}$	$a1 = a1 + a1 - a2$

Remember that the children have decided that zero represents the complete circle, rather than nothing. As we go back and look at the areas of the folded circle, there are in fact four areas to the circle. There are two sides to the circle divided into four individual areas of semicircles. We then have $a1$, $a2$, $a3$, and $a4$. They are all similar and also come in pairs. As AB = BA so does $a1 + a2 = a3 + a4$. Instead of two directions the areas have an inside and outside identified by folding the circle. By observing more about the circle there is more information, and coding can become very complex. We can always choose or not choose to include what makes sense to us. Keep it easy and clear, that is the point of using a code language.